Global Games and Finitely Repeated Networks: The Theory and Experimental Evidence

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Abstract

A finitely repeated two-side link network is analyzed. In order to select one of multiple equilibria that arise in this kind of setting, the global game approach is used as a notion of equilibrium. In the first stage of the game I restrict my analysis to particular kinds of strategies, called switching strategies. Under some restrictions on the payoff noise structure, players beliefs (and strategies) converge in the second stage of the game. This allows the game to be collapsed with a unique equilibrium which has several nice properties. For instance, it converges to the Harrison and Elbittar (2006) global game solution when the game is played once and it converges to the cooperative/efficient solution when the game converges to an infinitely repeated game. Experimental sessions testing the predictive power of the equilibrium are reported. The predictions of the model and comparative statics are consistent with the data. In the worst case, 75% of the strategies played by individuals in the experiment can be explained by the model.

1 Introduction

The increasing interest of economists in understanding social networks stems from the recognition that the economic context is an important determinant of many social phenomena. The study of networks have opened a door to understanding situations where the market is not the principal mechanism for the distribution of goods. Labour networks, collaboration among firms in R & D, political agreements in the congress and, the flow of private information are examples where networks rather than markets may better explain behavior.

The main concern of networks analysis is the multiple equilibria problem. The introduction of graph structures and the so-called "Myerson value" by Myerson (1977) led to the study of networks

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in a cooperative context. Later, the work of Aumann and Myerson (1988) recognized that different graph structures led to different retributions of the agents involved in the network. Nowadays, two branches of the study of networks can be identified. One is the cooperative branch which emphasizes the stability of the graph structure; and the noncooperative one, which emphasizes the incentives for individuals to be part of a network (see Jackson (2003, 2007)) for a good discussion of this issue and the role of networks in economics).

Cooperative networks concepts are based on the idea that individuals can coordinate group deviations from their initial situations. Underpinning this behavior is some sort of communication among individuals (in Myerson's technical words, "individuals are path connected".) Criticisms of this approach come from the argument that communication may only arise when the network is already formed. So, cooperative refinements may not be enough to explain how networks structures emerge. On the other hand, the noncooperative approach looks like a good alternative to explaining how networks are formed, but is unlikely to explain stability of the network in the long term. Part of this work shows that these two approaches are not necessarily in conflict.

Among the noncooperative concepts, the Global Game approach, introduced by Calrsson and van Damme (1993), has shown it self to be potentially very useful in settings where multiple equilibria arise. In few words, global games consist of turning some parameters of the game from common knowledge to private information and using the iterative elimination of interim dominated strategies to refine the set of Nash equilibria. Harrison and Muñoz (2006) show that this concept can be applied to the network framework and, moreover, in a general setting the set of equilibria is a singleton.

The focus of this paper is to try to use the global game approach in a context where networks interact a finite number of times and to test the model predictions under experimental sessions. My work could be framed in an incipient literature of dynamic global games where some authors have put emphasis on learning processes as new information becomes available (see Angeletos, Hellwig and Pavan (2007) for a good example) and some others concentrate their efforts on finding conditions of equilibrium uniqueness (see, for instance, Heidhues and Melissas (2006)), In my setting, despite of the fact that the networks interact repeatedly, the equilibrium is similar to the static one, though it depends on the number of periods that the networks interact.

The experimental evidence with respect Global Games is inconclusive. On the one hand Heinemann, Nagel and Ockenfels (2004) test the Global Games predictions in a coordination game of speculative attack and find that predictions are only correct under an incomplete information scheme. On the other hand Elbittar, Harrison and Muñoz (2007) test this approach in a network scheme and find predictions extremely consistent with the evidence in both complete and incomplete information contexts. Also, noncooperative equilibria have been tested in networks by Falk and Kosfeld (2003). They found that in games where the link could be established unilaterally the strict Nash equilibrium works very well but in games with bilateral links it doesn't.

In this paper I analyze a finitely repeated version of the Aumann-Myerson Consent Game which is a bilateral link network, i.e. both players must agree to be connected, however the link may be severed unilaterally. If players play essentially unique strategies in the first stage game and if the noise structure has bounded support, I find that beliefs over other player observations (and hence strategies) converge in the second stage of the game. This allows us to collapse all the game structure into the first stage and use an approach similar to a static global game. However, individuals are forward looking, in the sense that they consider future payoffs to choose strategies that lead to a unique equilibrium. Also, I test the model under experimental conditions where I find that, in the worst case, 75% percent of the individuals that participate in the experiment are well explained by the model.

Section 2 describes the static consent game and its Nash equilibria. Section 3 introduces two cooperative refinements and the global game refinement in order to provide background. The repeated game and the theoretical developments are in section 4. Section 5 presents the experimental evidence and section 6 concludes.

2 The Static Model

2.1 The Basic Benchmark: The Three Players Case[1]

Consider a static version of the Consent Game with three agents and complete information. Let $\mathscr{I} = \{1,2,3\}$ be the set of players and A_i the set of actions for player $i \in \mathscr{I}$ where $A_i = \left\{ (a_{ij})_{j\in\{\mathscr{I}\setminus i\}} \middle| a_{ij} \in \{1,0\} \right\}$, for instance the action $a_2 = (a_{21} = 1, a_{23} = 0) \in A_2$ is two's action if he attempts to establish a connection with player number one and not with player three[2]. Since this is a two side link formation model, a link will be formed if and only if both players try to form the link. Formally, a link between ij is formed if and only if $a_{ij} = a_{ji} = 1$.

Let $x \in \mathbb{R}^{++}$ be a variable that scale the level of profits of individuals, then the payoff function is defined as follows

$$\begin{aligned}\pi_i(a_i, a_{-i}, x) &= a_{ij}a_{ji}(x + a_{jk}a_{kj}\beta x) + a_{ij}(\alpha x - c) + \\ &\quad + a_{ik}a_{ki}(x + a_{kj}a_{jk}\beta x) + a_{ik}(\alpha x - c)\end{aligned}$$

where α and β are positive fixed parameters that determine the possible revenue that a player can get given x; and c represents the connection attempt cost, which is also fixed and positive. Note that no matter whether there is a link between two players, if one individual tries to establish a connection he will get $(\alpha x - c)$. Also, note that the formation of a link can only increase the total revenue of an individual and this payoff function is symmetric in the sense that player i gets the same expected reward if he offers a link to player j or if he offers it to player k.

Given this, the game is defined as the triple $G(x) = \left\langle \mathscr{I}, (A_i)_{i\in\mathscr{I}}, (\Pi)_{i\in\mathscr{I}} \right\rangle$.

2.2 The Set of Nash Equilibria

In order to obtain the best response function, lets analyze each element of the payoff function. The term $a_{ij}(\alpha x - c)$ represents player i's cost or benefit from attempting a link with player j. It is easy

[1] Taken from Harrison and Muñoz (2006)

[2] I use notation that is standard in the literature. a_{-i} to name the action vector of all players but i. In the same logic, $A_{-i} = \times_{j\in\{\mathscr{I}\setminus i\}} A_j$ is the action space for every individual except i. In the particular case of three players we have $A_{-i} = A_j \times A_k$. Lastly, I abuse notation and denote $a_i = \mathbf{1}$ in order to express $(a_{ij} = 1, a_{ik} = 1)$ and $a_i = \mathbf{0}$ to express $(a_{ij} = 0, a_{ik} = 0)$.

to see that if $x > \frac{c}{\alpha}$ this will always be positive and it will be a dominant strategy to try a connection. On the other side, the term $a_{ij}a_{ji}(x)$ arises only when a link between players i and j is formed. Since $x > 0$, this component is strictly positive. Lastly, the element $a_{ij}a_{ji}(a_{jk}a_{kj}\beta x)$ occurs when, given a link between i and k, there is also a link between k and j; intuitively, this component represents an indirect benefit for the "relationship" among other players. If we understand this network as a friendship relation (as Jackson and Wolinsky (1996) does) people not only get benefits of their relations, they also get benefits from the friends of their friends.

In the three players case, if every link is formed, the total revenue for each individual is $2((1+\alpha+\beta)x-c)$; thus if $x < \frac{c}{1+\alpha+\beta}$ is a dominant strategy to never offer a link. Is important, to determine the set of Nash Equilibria, to know when two individuals can support a link. This occurs when $(1+\alpha)x-c \geq 0$, that is, when $x \geq \frac{c}{1+\alpha}$. Given all these elements, it is easy to show that the best response function is:

- If $x \in \left(\infty, \frac{c}{1+\alpha+\beta}\right)$, then

$$BR_i(a_{-i}) = \left\{a_i = \mathbf{0} \quad \forall a_{-j} \in A_{-j}\right\}$$

- If $x \in \left[\frac{c}{1+\alpha+\beta}, \frac{c}{1+\alpha}\right]$, then

$$BR_i(a_{-i}) = \begin{cases} a_i = \mathbf{1} & \text{if } \ a_{-i} = \mathbf{1} \\ a_{ij} = 1, a_{ik} = 0 & \text{if } \ a_{ji} = a_{jk} = a_{kj} = 1,\ a_{ki} = 0 \\ a_{ij} = 0, a_{ik} = 1 & \text{if } \ a_{ji} = 0,\ a_{ki} = a_{jk} = a_{kj} = 1 \\ a_i = \mathbf{0} & \text{otherwise} \end{cases}$$

- If $x \in \left[\frac{c}{1+\alpha}, \frac{c}{\alpha}\right]$, then

$$BR_i(a_{-i}) = \begin{cases} a_{ij} = 1, a_{ik} = 1 & \text{if } \ a_{ji} = a_{ki} = 1 \\ a_{ij} = 1, a_{ik} = 0 & \text{if } \ a_{ji} = 1,\ a_{ki} = 0 \\ a_{ij} = 0, a_{ik} = 1 & \text{if } \ a_{ji} = 0,\ a_{ki} = 1 \\ a_{ij} = 0, a_{ik} = 0 & \text{if } \ a_{ji} = 0,\ a_{ki} = 0 \end{cases}$$

- If $x \in \left(\frac{c}{\alpha}, \infty\right)$, then

$$BR_i(a_{-i}) = \left\{a_i = \mathbf{1} \quad \forall a_{-j} \in A_{-j}\right\}$$

The best response function allows us to find the sets of NE, which is represented in Figure 1.

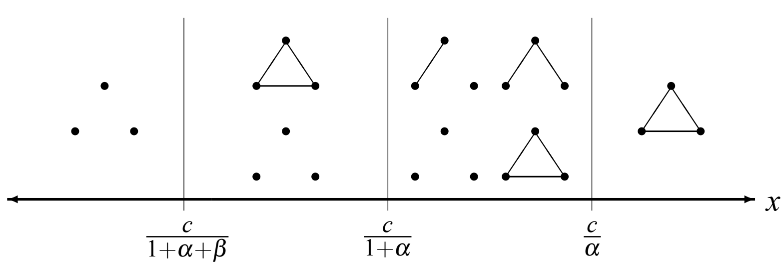

Figure 1 Networks structures supported by a Nash Equilibrium. Each agent is represented by a node and a link is represented by a line between nodes.

In Figure 1 each node represents an agent (any of them) and lines between nodes represent connections. The multiplicity of Nash Equilibria is evident. This leads us to think about how to establish a unique prediction. Next section introduces some refinements concepts.

3 Proposed Refinements

3.1 Cooperative Refinements

Cooperative refinements in networks are important when the focus of the analysis is properties of a network rather than individuals incentives. More precisely, these refinements say something about the stability of a network; Namely wether exist incentives for a network to change its form. Among the most influential cooperative concepts is “Pairwise Stability”, introduced by Jackson and Wolinsky (1996). They say that if no pair of individuals has incentives to do a join deviation, that is to sever or create a link, then the network is stable. Specifically, a Nash equilibrium is Pairwise Stable if the following two conditions hold:

i) If a link between two individuals is absent from the network then it cannot be that both individuals would benefit from adding the link (with at least one benefiting strictly.)

ii) If a link between two individual is present in a network the it cannot be that either individual would strictly benefit from deleting that link.

To have a better understanding of this concept let’s apply it to the set of Nash equilibria for the consent game. As Figure 2 shows, this set is refined; however is not strong enough to find a unique equilibrium.

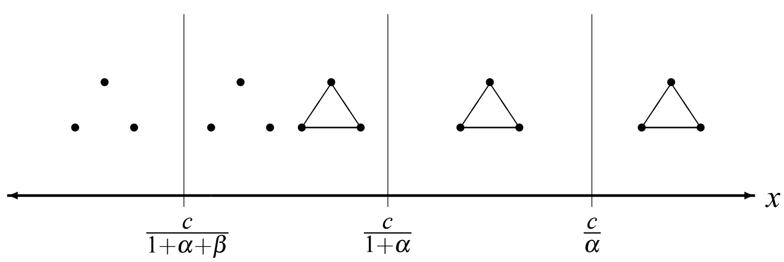

Figure 2 Network structures supported by a equilibrium Nash Pairwaise Stable.

The “weakness” of the previous concept comes from only allowing pairs of players to do a joint deviation. Other refinements with higher requirements have been developed in order to avoid the multiplicity. The Strong Stable Network introduced by Jackson and van den Nouweland (2005) permit coordinated deviations of groups of individuals and moreover, allows deviations such that individuals that are not part of it could be strictly worse off than the initial situation.

If we apply this stability notion to the Consent Game, the set of Nash equilibria turns to a singleton an the strategy played is a switching strategy with threshold at $x = \frac{c}{1+\alpha+\beta}$. That is, for all $x < \frac{c}{1+\alpha+\beta}$ nobody in the network tries to establish a connection with anybody; for all $x > \frac{c}{1+\alpha+\beta}$ everybody tries to connect with everyone and the complete network is formed. If $x = \frac{c}{1+\alpha+\beta}$, its not clear what will happen. For the last situation this strategy is called an *essentially unique* strategy. Figure 3 shows the networks structures supported by the Strong Stable Network notion.

Criticisms of this kind of approach come from the fact that to produce coordinated joint deviations of groups of individuals requires communication. This requirement is not too strong for individuals that are part of an existing network but if the network does not exist or individuals are outside the network, its very unlikely that this coordination can emerge. The next section analyzes this game under a non-cooperative scheme which is the basis of my work.

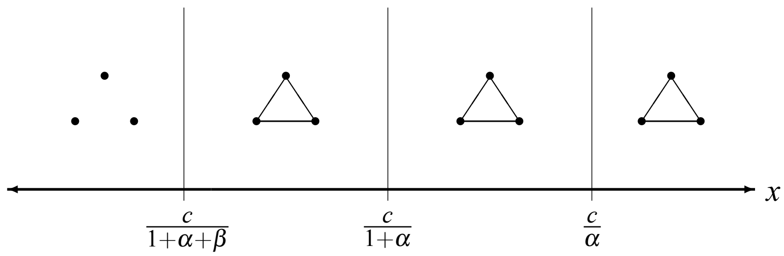

Figure 3 Network structures supported by a Strong Stable Network equilibrium.

3.2 The Global Game Refinement

The Global Game approach uses a Bayesian version of the previous model, hence it is a Bayesian Nash Equilibrium concept. In order to find it, we need to redefine some concepts and develop new ones. The next example is taken from Harrison and Muñoz (2006). Assume that there is a common value θ, which is draw uniformly from its support $\left[\underline{\theta},\overline{\theta}\right]$.[3] However, for each individual the level of possible revenues is scaled by $x_i = \theta + \sigma\varepsilon_i$, where σ is a weight factor and ε_i is the realization of a random variable with density ϕ and support in $\left[-\frac{1}{2},\frac{1}{2}\right]$. I assume that ε_i is *i.i.d.* across individuals.[4] This produces two categories of beliefs: player i's beliefs over θ, which are $\theta\,|x_i \sim U\left[x_i - \frac{\sigma}{2}, x_i + \frac{\sigma}{2}\right]$, and player i's beliefs over x_j (given its observation,) which are $x_j\,|x_i \sim U\left[x_i - \sigma, x_i + \sigma\right]$. A pure strategy for player i is a function from its private observation to its action space $s_i : \left[\underline{x} - \frac{\sigma}{2}, \bar{x} + \frac{\sigma}{2}\right] \to A_i$, the set of all such functions is denoted S_i. The pay off function is now

$$\begin{aligned}\pi_i\left(s_i, s_{-i}, x_i\right) &= a_{ij}a_{ji}(x_i + a_{jk}a_{kj}\beta x_i) + a_{ij}(\alpha x_i - c) + \\ &\quad + a_{ik}a_{ki}(x_i + a_{kj}a_{jk}\beta x_i) + a_{ik}(\alpha x_i - c)\end{aligned}$$

Call this game of incomplete information $G(\sigma)$.[5]

Let us define an *essentially unique* switching strategy between **0** and **1** with threshold k_i as a pure strategy $s_i(\cdot;k_i)$ satisfying:

$$s_i(x_i;k_i) = \begin{cases} \mathbf{1} & if \quad x_i > k_i \\ \mathbf{0} & if \quad x_i < k_i \end{cases}$$

Harrison and Muñoz (2006) prove the following proposition:

[3] I use the uniform distribution only for simplicity, the authors shown that the next analysis hold with any distribution

[4] Harrison and Muñoz develop their model in a more general payoff function context and their proofs also works if ε_i come from an *i.i.d.* process and it has a continuous distribution over $\mathbb{R}$

[5] This game corresponds to a private value definition. There is a common value definition where each player's pay-off function depends of θ instead of x_i

Proposition 1 *For any $\sigma > 0$, the essentially unique switching strategy profile $s^* = (s_i(\cdot;k^*))_{i\in\mathscr{I}}$ is the only strategy profile that survives iterated elimination of strictly dominated strategies in $G(\sigma)$, where $k^* = \frac{4c}{2+4\alpha+\frac{4}{3}\beta}$.*

Figure 4 shows the set of Network structures supported by BNE for the consent Game. As can be seen, there is also a switching point but it is totally different to the ones predicted by cooperative equilibria.

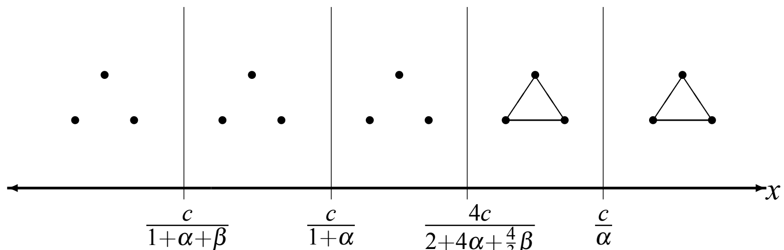

Figure 4 Networks structures supported by BNE.

Harrison and Muñoz (2006) extend these results in settings with finite numbers of players and more general payoff functions. Elbittar *et al.* (2007) develops experimental sessions testing this game and find that global game predictions are extremely consistent with the data.

4 The Repeated Game

4.1 The Model

I now introduce a *repeated* version of the Consent Game and incomplete information. Consider a set $\mathscr{I} = \{1,2,\ldots,I\}$ of players and let $\mathscr{T} = \{1,2,...,T\}$ be the number of periods that the game is repeated where T is finite. Define $A_i^t = \left\{ \left(a_{ij}^t\right)_{j\in\{\mathscr{I}/i\}} \middle| a_{ij}^t \in \{0,1\} \right\}$ as the action space of individual i in period t. Assume that there is a common value θ, which is drawn uniformly[6] from its support $\left[\underline{\theta},\overline{\theta}\right]$. However, for each individual the level of possible revenues is scaled by $x_i = \theta + \sigma\varepsilon_i$, where σ is a weigh factor and ε_i is the realization of a random variable with continuous density $f(\cdot)$ and support in a bounded interval of $\mathbb{R}$. I assume that ε_i is *i.i.d.* across individuals.

Let $a_i^t = \left(a_{i-i}^t\right)$ be the vector of actions of player i at period t and a^t the vector of actions of all players at period t. We can define the *observed* history of the game as follow:

$$h_{ij}^t = \left\{ \begin{array}{cc} a_j^t & if\ a_{ij}^t = 1 \\ \emptyset & if\ a_{ij}^t = 0 \end{array} \right\} \cup h_{ij}^{t-1},\ with\ h_{ij}^1 = \emptyset$$

that is, player i can observe player j's action if and only if player i tries to establish a connection with player j. Note that this object is defined recursively.

[6]As the previus section, I use the uniform distribution only for simplicity, it can be shown that the next analysis hold with any distribution

A strategy for player i at t is a function from his private value and observed history to his action space, that is $s_i^t : (x_i, h_i^t) \rightarrow A_i^t$. Finally, the payoff function is

$$\pi_i(s_i, s_{-i}, x_i) = \sum_{t=1}^{T} \pi_i^t(s_i^t, s_{-i}^t, x_i) \text{ where}$$

$$\pi_i^t(s_i^t, s_{-i}^t, x_i) = \sum_{j \in \{\mathscr{I} \setminus i\}} \left\{ a_{ij}^t a_{ji}^t \left(x_i + \sum_{k \in \{\mathscr{I} \setminus \{i,j\}\}} a_{jk}^t a_{kj}^t \beta x_i \right) + a_{ij}^t (\alpha x_i - c) \right\}$$

Call this game of incomplete information the $\mathscr{T} - G(\sigma)$.

In the first stage of the game I will restrict my analysis to *essentially unique* switching strategies between **0** and **1** with threshold k, that is:

- If player i observes $x_i \geq k$, then he plays $s_i = \mathbf{1}$.
- If player i observes $x_i \leq k$, then he plays $s_i = \mathbf{0}$.

The previous restriction allows me to write the payoff function in period 1 as:

$$\pi^1(s_i^1, l_i, x_i)$$

where l_i is the number of players other than i that try to establish a connection at period 1. In other words, revenues depend on the graph structure of the network. That is, the payoff function satisfies the anonymity principle.

The next subsections are devoted to characterizing the beliefs at each stage of the game, which is an important part of the equilibrium description. I will then find the equilibrium and its properties.

4.2 The Description of Beliefs

The characterization of beliefs is an essential part of the equilibrium description. Here three elements have an important role:

1. Player i's beliefs about θ given x_i,
2. player one's beliefs about x_{-i} given x_i and
3. the (Bayesian) update of the beliefs when new information arrives.

As I am going to show, the first element is important only in the first stage of the game. The third element is important from the second state and above and the second element is always important.

4.2.1 The First Stage Beliefs

Beliefs over θ From the model description we know that each player observes the signal $x_i = \theta + \sigma\varepsilon_i$, where σ is a scale factor and ε_i is the realization of an *i.i.d* random variable with continuous density $f(\cdot)$ and support in a bounded interval of $\mathbb{R}$. Then, given that player i has the signal x_i, he assign the probability $f\left(\frac{x_i-\theta}{\sigma}\right)$ to the state of nature θ.

Beliefs over l_i^1 In the context of switching strategies, the previous distribution allows me to describe the probability that player i assign to l players observing a private signal x_j above threshold k and, therefore, trying to establish a connection. This probability is:

$$\int f\left(\frac{x-\theta}{\sigma}\right)\binom{I}{I-l}\left[F\left(\frac{k-\theta}{\sigma}\right)\right]^{I-l}\left[1-F\left(\frac{k-\theta}{\sigma}\right)\right]^{l}d\theta$$

where $\left[F\left(\frac{k-\theta}{\sigma}\right)\right]^{I-l}$ is the probability that $I-l$ players observe a signal below k and $\left[1-F\left(\frac{k-\theta}{\sigma}\right)\right]^{l}$ is the probability that l players observe a signal above k. Finally $\binom{I}{I-l}$ is the combinatorial of players that can satisfy the previous requirement. Given this, the expected payoff for the first stage of the game is:

$$\sum_{l=0}^{I-1}\pi\left(s_i^1,l,x_i\right)\int f\left(\frac{x-\theta}{\sigma}\right)\binom{I}{I-l}\left[F\left(\frac{k-\theta}{\sigma}\right)\right]^{I-l}\left[1-F\left(\frac{k-\theta}{\sigma}\right)\right]^{l}d\theta$$

4.2.2 Beliefs for $t \geq 2$

Since in the first period of the game we use the distribution of θ to get the beliefs of how many players observe its private observation above the threshold point, in the second period of the game the distribution of θ lost its importance in the analysis. This is because the main concern of player i is to know if player j will offer connection in the second stage but, from player j actions in period one, player i will know if x_i is above or below k_i. More precisely, recall that player i observes player j actions if and only if $a_{ij} = 1$, recall also that, in $t = 1$, individuals play only essentially unique switching strategies. Therefore, there only exist two possibilities: player one plays $a_i^1 = \mathbf{1}$ and observes a^1; or player i plays $a_i^1 = \mathbf{0}$ and does not observe anything. In the case that player i observe a^1 he will know exactly which players have an $x_i \geq k_i$. Also, is important to note that every player who has an $x_i \geq k_i$ discovers in $t = 2$ all other players with observation above the thresholds point and this is common knowledge among players. Using all these elements I am going to prove the following proposition.

Proposition 2 *Let $\mathcal{T}-G(\sigma)$ be defined as above. Then for sufficiently small σ beliefs over other player actions converge in the second stage of the game.*

Proof. In order to prove this proposition we have to analyze three cases. First, note that if a player gets an $x_i \leq k_i$ he offers no links and he gets no new information about other player actions, so he has no beliefs update either. Then, if in period one his observation does not survive to the iterative elimination of interim dominated strategies, his observation will not survive in the second stage either and he will offer no connections (also note that the argument holds for later periods too). The first case is when player i plays $a_i^1 = \mathbf{1}$ and all other players do not try to connect, that is $a_{-i}^1 = \mathbf{0}$. If this is the case, player i beliefs about other players actions are $\Pr_i^{t\geq 2}\left(a_{-i}^t = \mathbf{0}\,|h_i^t, x_i\right) = 1$. The second case is when player i plays $a_i^1 = \mathbf{1}$ and all other players do connect, that is $a_{-i}^1 = \mathbf{1}$. If this is the case, player i beliefs of other players actions are $\Pr_i^{t\geq 2}\left(a_{-i}^t = \mathbf{1}\,|h_i^t, x_i\right) = 1$. The last and more complex case is when some players other than i offer connections but not all of them. Recall from the description of Nash equilibrium that a player does not want to serve a connection in an scenario where the complete network is not formed if and only if $x_i \geq \frac{c}{1+\alpha}$. The question that arises in this contexts is, given that player i observes $x_i \geq \frac{c}{1+\alpha}$, what is the probability of the connected players have an $x_j \geq \frac{c}{1+\alpha}$? For games with sufficiently small σ and ε_i with bounded support this probability is 1**.** To see this look at Figure 5. The limits of the support of player i beliefs over other player observations is a function of σ, it is possible to find $\underline{\sigma_i}$ such that for all $\sigma \leq \underline{\sigma_i}$ the support of player i beliefs over other player observation is always above $\frac{c}{1+\alpha}$. Then if we take the minimum σ_i among players the statement holds for all players and $\Pr_i^{t\geq 2}\left(a_{ji}^t = 1\,\middle|h_i^1, x_i\right) = 1$, where j is a player that offered connection in the first period. In the case where $x_i < \frac{c}{1+\alpha}$ player i wants so sever the connection, and for sufficiently small σ every player want to do the same. ■

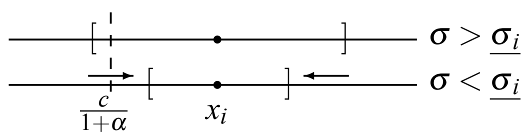

Figure 5: Support of the beliefs of other player private value for a player observing $x_i \geq \frac{c}{1+\alpha}$ for different σ.

The previous proposition allow us to state:

Proposition 3 *Let $\mathcal{T} - G(\sigma)$ be defined as above. Then, for sufficiently small σ, each profile S^1 has associated only one profile S^t for $t \geq 2$, such that it is the best response to each player beliefs.*

Proof. For regions of x_i where a dominant strategy exists, the result trivially holds. If player i plays $a_i^1 = \mathbf{0}$, he has no new information and we will play $a_i^t = \mathbf{0}$ for all t. If player i plays $a_i^1 = \mathbf{1}$ and he observes that $a_{-i}^1 = \mathbf{0}$, then he will assign $\Pr_i^{t\geq 2}\left(a_{-i}^t = \mathbf{0}\,\middle|h_i^1, x_i\right) = 1$ and he will play $a_i^t = \mathbf{0}$ for all $t \geq 2$. If player i plays $a_i^1 = \mathbf{1}$ and he observes that $a_{-i}^1 = \mathbf{1}$, then he assign $\Pr_i^{t\geq 2}\left(a_{-i}^t = \mathbf{1}\,\middle|h_i^1, x_i\right) = 1$ and he will play $a_i^t = \mathbf{1}$ for all $t \geq 2$. If player i plays $a_i^1 = \mathbf{1}$**,** the complete network is not formed and $x_i \geq \frac{c}{1+\alpha}$, then for connected players he will believe that $\Pr_i^{t\geq 2}\left(a_{ji}^t = 1\,\middle|h_i^1, x_i\right) = 1$ and his best response is to connect to those players and no to connect with players that have-not offer a link in the previously period. If player i plays $a_i^1 = \mathbf{1}$ and $x_i < \frac{c}{1+\alpha}$, then his best response is not to offer any connection. ■

Corollary 4 *Strategies in $t \geq 2$ for individuals that offered connection in $t = 1$, can be represented as a function from l_1 and x_i, that is:*

$$s_i^t : (l_1, x_i) \rightarrow A_i^t$$

Proof. This function is

$$s_i^t = \begin{cases} if\ l_1 = 0 & then\ s_i^t = \mathbf{0} \\ if\ l_1 = I-1 & then\ s_i^t = \mathbf{1} \\ if\ 0 < l_1 < I-1 \text{ and } x_i < \frac{c}{1+\alpha} & then\ s_i^t = \mathbf{0} \\ if\ 0 < l_1 < I-1 \text{ and } x_i \geq \frac{c}{1+\alpha} & \begin{cases} if\ a_j^1 = \mathbf{0} \rightarrow a_{ij} = 0 \\ if\ a_j^1 = \mathbf{1} \rightarrow a_{ij} = 1 \end{cases} \end{cases}$$

stating the corollary. ■

This corollary is very useful, since allows us to describe all the strategies profiles for $t \geq 2$ as a function of l_1. Therefore, payoffs of all periods are fully characterized by l_1. I am going to use this to find the equilibrium in the next section.

4.3 The Global Game Refinement

The equilibrium notion developed in this section closely follow Global Games. The main difference is that in this setting a repeated game is played. However, at is was shown in the last section, game payoffs and strategies are fully determined in the first period of the game. So, we can use the static global game approach to find the equilibrium.

Let's define $\Delta\pi_i(l_1,x_i) = \pi_i(1,l_1,x_i) - \pi_i(0,l_1,x_i)$. Note that $\pi_i(0,l_1,x_i) = 0$ for all l_1 and x_i, hence $\Delta\pi_i(l_1,x_i) = \pi_i(1,l_1,x_i)$. With this we can state the next proposition.

Proposition 5 *The payoff function satisfies the following properties*

P5.1 ***Strategic Complementarities:*** $\Delta\pi(l_1,x_i)$ *is increasing in* l_1.

P5.2 ***State Monotonicity:*** $\Delta\pi(l_1,x_i)$ *is increasing in x.*

P5.3 ***Single Crossing:*** *There is a unique* k^* *solving* $\sum_{l=0}^{I-1} \frac{1}{I}\Delta\pi(l,k) = 0$.

P5.4 ***Limit Dominance:*** *There exist* $\underline{x} \in \mathbb{R}^{++}$ *and* $\overline{x} \in \mathbb{R}^{++}$ *such that* $[1]\,\Delta\pi(l_1,x) < 0$ *for all* l_1 *and* $x \leq \underline{x}$*; and* $[2]\,\Delta\pi(l_1,x) > 0$ *for all* l_1 *and* $x \geq \bar{x}$.

P5.5 ***Continuity****:* $\sum_{l=0}^{I-1} g(l)\,\Delta\pi(l_1,x)$ *is continuous with respect to signal x and density g.*

The proof is relegated to the appendix A. These properties are very standard in the Global Game literature, see Morris and Shin (2002) or Frankel et al. (2003), and give us sufficient tools to prove the next theorem, which is the main result of this work.

Theorem 6 *Let* $\mathscr{T} - G(\sigma)$ *be defined as above. Then , for sufficiently small* σ*, the set of essentially unique switching strategies with threshold at k is singleton, where k satisfies:*

$$\sum_{l=0}^{I-1} \frac{1}{I}\Delta\pi(l,k) = 0 \tag{1}$$

and is the only strategy that survives to the iterative elimination of strictly dominated strategies.

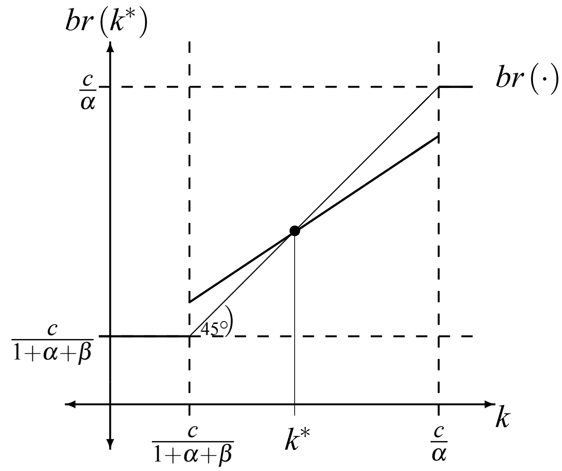

Figure 6: The best response function is a continuous function from the compact set $[\underline{x}, \bar{x}]$ to it self, then a fixed point exists which is represented by the point where the two sequences of the proof converges.

Proof. This proof has two steps. The first is to prove that the set of strategies is singleton, the second is to prove that the threshold point is defined by (1). **Step one:** By symmetry all players have the same threshold point, i.e. $k_i = k$. Let's define $\pi^*(x,k)$ as the expected utility of a player that offer connection to every one, observe x and other players plays the threshold k.

$$\pi^*(x,k) = \sum_{l=0}^{I-1} \int f\left(\frac{x-\theta}{\sigma}\right) \binom{I}{I-l} \left[F\left(\frac{k-\theta}{\sigma}\right)\right]^{I-l} \left[1-F\left(\frac{k-\theta}{\sigma}\right)\right]^{l} d\theta \pi(1,l,x)$$

note that by P5.1 and P5.2 the previous definition is strictly increasing in x and strictly decreasing in k. With this, we can define two sequences

$$\begin{array}{c} \overline{\xi}_{n+1} = \min\left\{x : \pi^*\left(x, \overline{\xi}_n\right) = 0\right\} \text{ and } \overline{\xi}_0 = \bar{x} \\ \underline{\xi}_{n+1} = \max\left\{x : \pi^*\left(x, \underline{\xi}_n\right) = 0\right\} \text{ and } \underline{\xi}_0 = \underline{x} \end{array}$$

the first sequence represent the iterative elimination of strictly dominated strategies starting from the upper dominance region and the second represent the same concept from the lower dominance region. Note, for the same argument exposed above, that the first sequence is strictly decreasing and the second is strictly increasing. By construction, the sequences satisfy $\overline{\xi}_{n+1} \geq \underline{\xi}_{n+1}$ for all n, which means that are two monotone sequences defined in a bounded interval, then both sequences must converge. Moreover, the must converge to the same point, hence the equilibrium is unique. To prove the last claim let's call $\lim_{n\to\infty} \overline{\xi}_{n+1} = \overline{\xi}$ and $\lim_{n\to\infty} \underline{\xi}_{n+1} = \underline{\xi}$. Take an $x \in \left(\underline{\xi}, \overline{\xi}\right)$, by continuity (P5.5) and the definition of $\underline{\xi}$ and $\overline{\xi}$ implies $\pi^*\left(x, \underline{\xi}\right) > 0$ and $\pi^*\left(x, \overline{\xi}\right) < 0$, respectively, which is an contradiction, so $\overline{\xi} = \underline{\xi}$. In other words, when $x = k$

$$\pi^*(x,x) = \sum_{l=0}^{I-1} \int f\left(\frac{x-\theta}{\sigma}\right) \binom{I}{I-l} \left[F\left(\frac{x-\theta}{\sigma}\right)\right]^{I-l} \left[1-F\left(\frac{x-\theta}{\sigma}\right)\right]^{l} d\theta \Delta\pi(l,x) = 0$$

Second Step: I have to show that

$$\int f\left(\frac{x-\theta}{\sigma}\right) \binom{2}{2-l} \left[F\left(\frac{k-\theta}{\sigma}\right)\right]^{2-l} \left[1-F\left(\frac{k-\theta}{\sigma}\right)\right]^{l} d\theta = \frac{1}{I}$$

when $x=k$. For this just take the the integral by parts and the result holds. So, we finally get

$$\sum_{l=0}^{I-1} \frac{1}{I}\Delta\pi(l,x)=0$$

which exist by P5.3. Figure 6 shows a picture of the intuition of this result ■

4.4 Properties of the Equilibrium for three Players

This section characterizes some properties of the equilibrium for the three player case. Motivations for this are simplicity (our benchmark is constructed under a three player scheme,) tractability and developing the theoretical findings that are tested in the experimental section. Appendix A show that all this results can be extended to the I players case.

Corollary 7 *The threshold point k in a game with three players and length T is:*

$$k^*(T)=\left\{\begin{bmatrix} \frac{2c(2+T)}{(2(T)+1)+2\alpha(2+T)+2(T)\beta} & if\ \frac{3}{2}\beta\leq T \\ \frac{3c(1+T)}{3T+3\alpha(1+T)+2T\beta} & if\ \frac{3}{2}\beta\geq T \end{bmatrix}\right.$$

Proof. From proposition 6 we know

$$\sum_{l=0}^{2} \frac{1}{3}\Delta\pi(l,x)=0$$

Its important to note that strategies in $t\geq 2$ are conditioned to which interval is k^*. If $k^*\in\left[\frac{c}{1+\alpha+\beta},\frac{c}{1+\alpha}\right]$ equation (1) becomes:

$$\frac{1}{3}2T\left[(1+\alpha+\beta)x-c\right]+\frac{1}{3}\left[(1+2\alpha)x-2c\right]+\frac{1}{3}2\left[\alpha x-c\right]=0$$

and implies

$$k(T)^*=\frac{2c(2+T)}{(2T+1)+2\alpha(2+T)+2T\beta}$$

If $k^*\in\left[\frac{c}{1+\alpha},\frac{c}{\alpha}\right]$ equation (1) becomes:

$$\frac{1}{3}2T\left[(1+\alpha+\beta)x-c\right]+\frac{1}{3}\left[T\left((1+\alpha)x-c\right)+\alpha x-c\right]+\frac{1}{3}2\left[\alpha x-c\right]=0$$

and implies

$$k(T)^*=\frac{3c(1+T)}{3T+3\alpha(1+T)+2T\beta}$$

It can be easily shown that the final solution is:

$$k^*(T)=\left\{\begin{bmatrix} \frac{2c(2+T)}{(2(T)+1)+2\alpha(2+T)+2(T)\beta} & if\ \frac{3}{2}\beta\leq T \\ \frac{3c(1+T)}{3T+3\alpha(1+T)+2T\beta} & if\ \frac{3}{2}\beta\geq T \end{bmatrix}\right.$$

■

Since my approach is close to Global Games, it is desirable to both solutions coincide if $T = 1$. The next corollary state this result.

Corollary 8 *When $\mathscr{T} - G(\sigma)$ has three players and $T = 1$, the solution converges to the Global Game solution.*

Proof. Just take $T = 1$,

$$k^*(T) = \left\{ \begin{bmatrix} \frac{4c}{2+4\alpha+\frac{4}{3}\beta} & si\ \frac{3}{2}\beta \leq T \\ \frac{4c}{2+4\alpha+\frac{4}{3}\beta} & si\ \frac{3}{2}\beta \geq T \end{bmatrix} \right.$$

which is the static solution. ■

Harrison and Muñoz (2006) show that in a set of payoff functions satisfying a weaker version of proposition 5 the set efficient solution, i.e. where the summation over all players payoff is maximized, is a switching strategy at the lower dominance point, in this case $\underline{x} = \frac{c}{1+\alpha+\beta}$. Is interesting to see that, in the limit, when the repeated game converges to a infinitely repeated game, the noncooperative switching point is the efficient solution.

Corollary 9 *When $\mathscr{T} - G(\sigma)$ converges to an infinitely repeated game, the optimal strategy is to play the efficient/cooperative solution.*

Proof. Take the limit when $T \to \infty$

$$\lim_{T\to\infty} k^*(T) = \left\{ \begin{bmatrix} \frac{c}{1+\alpha+\beta} & si\ \frac{3}{2}\beta \leq T \\ \frac{c}{1+\alpha+\frac{2}{3}\beta} & si\ \frac{3}{2}\beta \geq T \end{bmatrix} \right.$$

However, if $T \to \infty$ only the case $\frac{3}{2}\beta \leq T$ holds, so $\lim_{T\to\infty}$

$$k^*(\infty) = \frac{c}{1+\alpha+\beta}$$

which is the efficient solution. ■

This result has an especial importance since, in certain way, we are finding a noncooperative foundation for cooperative notions of equilibrium and, moreover, this behavior is efficient.[7]

The purpose of the next section is to explain the experimental session designed to test the predictability of the previous model.

[7]However, its not clear for the author if this is just a feature of this setting, a coincidence or if there is a general principle that can be proved

Table 1: Experimental Design

Treatment	Length (T)	Noise (σ)	Number of Sessions
1	5	0	1
2	5	10	1
3	1	0	2

Note 1: In all these sessions $\alpha = 0.4$, $\beta = 0.2$ and $c = 120$

5 Experimental Evidence

The experimental design was constructed based in the previous framework. Where three individuals that they do not know each other have to decide if offer or not connection to the other two members of the group. Elbittar et all (2007) showed that, in the static game, the Global Game approach describes in an accurate way networks behavior. The main goal of this section is to test if the repeated version works as good as the static concept in a similar setting.

Experimental sessions were designed with different noise structures (σ) and different lengths (T). However, payoff structures (α, β and c) are common in all treatments. The purpose of this design is to capture comparative static effects for this game. Table 1 summarizes different treatments in the experiment. Next section describes the experimental procedure.

5.1 Experiment Description

This section describes the general experimental procedure.

Participants and Venue. Subjects were drawn from a wide cross–section of undergraduate and graduate students at Pontificia Universidad Católica de Chile (PUC) in Santiago, Chile. Each subject participated in only one session. The experiment was run using computers.

Number of Periods. In order to familiarize subjects with the procedures, ten practice periods (grouped in ten games of length 1 or two games of length 5) were conducted in each session before the fifty real (affecting monetary payoff) periods (grouped in fifty games of length 1 or ten games of length 5.)

Matching Procedure and Group Size. At the beginning of each game, the computer randomly formed groups of three participants, so that each participant formed part of a new group in each of the following games. Furthermore, participants did not know who they were grouped with in any given game. In each session there were 15 participants.

Link Procedure and Payoff Structure. All participants were informed that each of them have to decide in every period whether to request a link with zero, one, or two members in her group. They were explained that a participant monetary payoff for each link request she decide to make would be the sum of the following three components: *i*) Unilateral connection component ($= \alpha \times x - c$): It is the payoff a participant would get if she request a link with another participant. *ii*) Complete connection component ($= x$): It is the payoff a participant would get if the link is formed, in other words, if both parts request the link. *iii*) Indirect connection component ($= \beta \times x$): It is the payoff a participant would get if the agent she has established a link has also established a link with the third agent of the group. Finally, they were also informed that in case a participant has decided do not make any link request, her monetary payoff would be zero for that period.

Valuation Distribution. For the complete information setup, all participants were informed that every member of her group would receive in every game a connection value, x, which remain constant in every period for game with length over than one, generated randomly by the computer from the interval 50.00 to 310.00 and any value in this interval would have an equally likely chance of being draw. For the incomplete information setup, all participants were informed that every member of her group would privately receive in every period a connection value, x_i, generated by the computer in the following manner: i) In each period and for every group, a number, x_0, would be drawn randomly from the interval 50.00 to 310.00 and any value in this interval would have an equally likely chance of being draw. ii) Once x_0 was determined, each group member would receive a private connection value, x_i, independently selected within the interval $[x_0 - \sigma, x_0 + \sigma]$, such that any value in this interval would have an equally likely chance of being draw. Furthermore, participants did not know the value of x_0 nor the private value of any other participant in their group, unless the value of σ were zero for the session.

Parameter Values. All participants were informed at the beginning of the session the values for alpha (α), beta (β), connection cost (c), sigma (σ) and game length (T.)

Minimum Capital and Payoff Procedure. Each participant received an initial points balance of 9,000 points. Participants were paid the total points earned (or lost) from each decision period, plus the initial point balance, multiplied by $0.33 Chilean pesos per point at the end of the experiment. The average payoff per participant for the whole session was $5,250 Chilean pesos (about 11.0 US dollars).

Information Feedback. In every period, each participant observed only her own payoff coming from each of the other members of her group and discriminated by each of the three payoff components already mentioned. Communication among the participants was not allow throughout all sessions. They could not see each others' screens.

5.2 Experimental Results

The first element tested was the restriction of focusing only in essentially unique switching strategies at period one. Using a Bernoulli distribution in order to describe each player action, the probability of observing a switching strategy has a binomial distribution.

Result 1 *At most* the probability of observing a non essentially unique strategy in the first stage is 13%.

Table 2 shows statistics and confidence intervals for this test in all treatments. As can be seen, we can reject the hypothesis that only essentially unique strategies where played. However, the proportion of those strategies is very high, at least around the 90% of the strategies played is essentially unique.

Table 2: Statistics for non-essenially unique strategies

Treatment	Session	Mean	Variance	Confidence Interval*
$T=5,\sigma=0$	1	2.67%	0.0173%	$[0.09\%,5.24\%]$
$T=5,\sigma=10$	1	10.00%	0.0600%	$[5.20\%,14.80\%]$
$T=1,\sigma=0$	1	6.93%	0.0086%	$[5.12\%,8.27\%]$
$T=1,\sigma=0$	2	12.00%	0.0141%	$[9.67\%,12.89\%]$

*95% of confidence

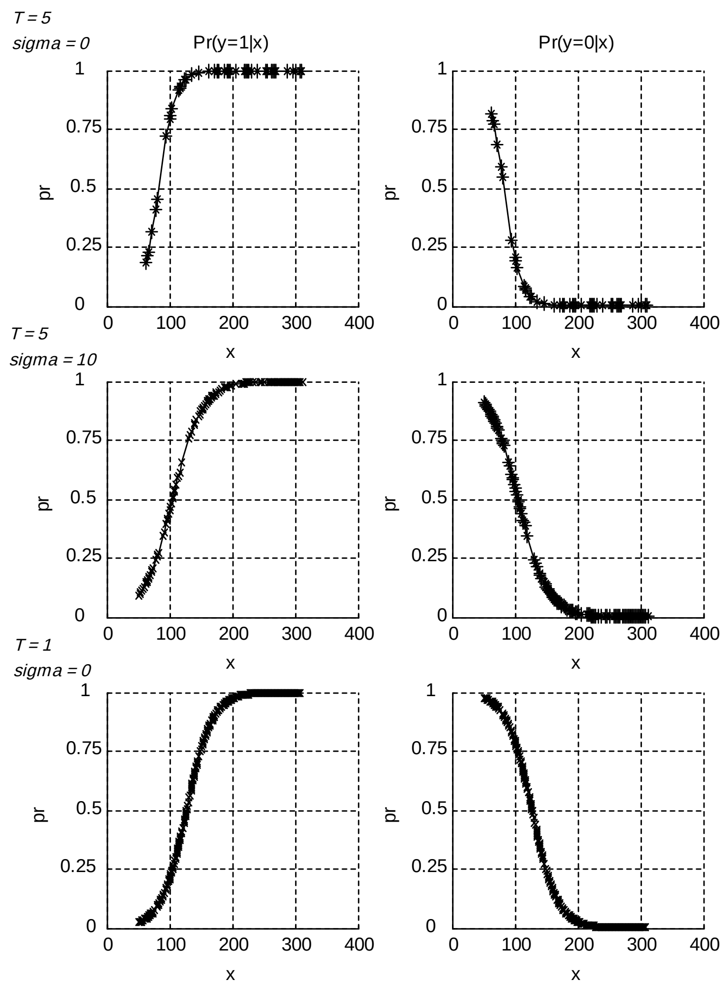

Figure 7: Accumulative link request probability for different treatments.

The next step in the analysis is to determine which is the threshold point of the switching strategies. A Logit model was used with the purpose to find the probability of offer connection with all

Table 3: Logit Model Result for Individual Link Request Probability for different game lenght

	Link Request Probability		
Coefficients	$T=1,\sigma=0$	$T=5,\sigma=0$	$T=5,\sigma=0$
$Intercept$	-6.243*	-6.185*	-4.461*
	(.2523)	(0.965)	(.6338)
x	.0491*	.0751*	.0428*
	(.0019)	(1.096)	(.0054187)
Number of Obs.	3000	300	300
Number of Ind.	30	15	15
Log Likelihood	-729.34338	-47.7	-77.6

* $p< 0.001$. Note: The number in parentheses below each coefficient represent the coefficient standard error.

Table 4: Statistics for Switching Thresholds

Treatment	Session	$\underline{x}$	k^*	$\hat{k}$	Interval*
$T=5,\sigma=0$	1	75	90	82	$[62,122]$
$T=5,\sigma=10$	1	75	90	105	$[84,140]$
$T=1,\sigma=0$	1	75	124	126	$[116,135]$
$T=1,\sigma=0$	2	75	124	120	$[114,134]$

*95% of confidence

individuals:[8]

$$\Pr\left(s_i^1=\mathbf{1}\,|x_i\right)=G\left(\gamma_0+\gamma_1 x_i\right),\ where\ G(z)=\frac{e^z}{(1+e^z)}$$

The results of these regressions are summarized in Table 3. As it can be seen, all parameters are significant at 1%. Figure 7 shows the accumulative distribution of both strategies, $s_i^1=1$ in the first column and $s_i^1=0$ in the second column.

Result 2 In all treatments the predicted switching point is in the interval of confidence.

To find the switching point I look at the point where $\Pr\left(s_i^1=\mathbf{1}\,|x_i\right)=\Pr\left(s_i^1=\mathbf{0}\,|x_i\right)=50\%$. Table 4 shows the starting point of the lower dominance region ($\underline{x}$,) the predicted switching point (k^*,) the estimated switching point ($\hat{k}$) and its confidence interval for each treatment. As can be seen, estimated parameters change according the theory, furthermore they always are in the confidence interval.

The last element tested is the convergence of s_i^t for $t\geq 2$. With this purpose two measurements were created. The first one is a binomial distribution where the random variable takes the value of $y=1$ if the profile s_i^t changes after the second period and $y=0$ if it do not change. With this measure I

[8] I also used a Multinomial Logit and a Ordered Logit. All methods have its advantage and problems. However, regressions were extremely robust. The three models predicts almost the same switching point in each treatments.

Table 5: Statistics for Switching Thresholds

Treatment	Session	Mean	Standard Deviation	Confidence Interval*
		Among Profiles		
$T = 5, \sigma = 0$	1	23.33%	1.99%	$[19.43\%; 27.24\%]$
$T = 5, \sigma = 10$	1	27.22%	3.64%	$[20.20\%; 34.46\%]$
		Within Profiles		
$T = 5, \sigma = 0$	1	12.22%	1.54%	$[9.20\%, 15.25\%]$
$T = 5, \sigma = 10$	1	14.88%	1.68%	$[11.59\%; 18.17\%]$

*95% of confidence

capture the essence of proposition number 3. The second one is also a binomial distribution where the random variable takes the value of $y = 1$ if $s_i^t \neq s_i^{t-1}$ for $t \geq 3$ and $y = 0$ if $s_i^t = s_i^{t-1}$ for $t \geq 3$. This second measure shows the frequency of change inside a profile. If all strategies change in every period then this measure takes the value of 1 and if it never change, takes the value of 0.

Result 3 At least the 75% of the strategies profiles converges.

As Table 5 shows, the first measurement is around 25%. This implies that a fourth of the strategies does not converge in the second stage. However, the variation inside strategies is low. This lead us to think that strategies converge in later stage of the game.

6 Conclusions

Despite off the fact that this game collapse in the first stage, it is interesting to remark that the final outcome depends on the game length. As greater the number of time that the stage game is repeated the lower will be the switching point. The relevance of this result increase when the limit when T goes to infinity is taken because the cooperative/efficient solution arises. This result could be understand as a bridge between the cooperative and the noncooperative literature in networks which remained unrelated up to this point. Further efforts could be done in this area in order to determine which are the determinant factors of this result.

Under the experimental point of view, the fact that at least the 75% of the population that participated in the experiment is explained by the theoretical framework described in this paper add more evidence to the potential of the global game approach. However, since I have few sessions and treatments this evidence should be taken carefully.

Further research could focus on drop main assumptions of this work like x_i remains constant throw the stage of the game or use a more general payoff function and study some applications. Under my point of view, a productive field could be industrial organization where this framework can be applied to understand market structures or asymmetries between the entrance or the exit of a firm in market.

A Appendix: Omitted Proofs

A.1. Proof of Proposition 5

1. **Strategic Complementarities:** Holds since α and β are positive.
2. **State Monotonicity:** Just take the derivative.
3. **Limit Dominance:** I found it in section 2 and their are $\underline{x} = \frac{c}{1+\alpha+\beta}$ and $\bar{x} = \frac{c}{\alpha}$.
4. **Continuity**: Since $f(\cdot)$ is continuos, $g(\cdot)$ is continuous too. $\Delta\pi$ is continuos, since π is continuos. And the composition of continuos functions is continuos.
5. **Single Crossing:** Since $\Delta\pi(l,x_i)$ is continuous in x and exists limit dominance. Then, for central theorem of calculus must exist an x such that $\sum_{l=0}^{2} \frac{1}{I}\Delta\pi(l,x) = 0$■

A.2. Equilibrium Properties for I Players.

Analytic Solution for T Periods and I Players

The payoff of offer connection to everyone, given x_i, and l players distinct that i get connected is.

$$\pi_i(1,l,x_i) = l\,(x_i + (l-1)\,\beta x_i) + (I-1)\,(\alpha x_i - c)$$

So, for $k \in \left[\frac{c}{1+\alpha+\beta}, \frac{c}{1+\alpha}\right]$, equation (1) is:

$$\sum_{l=0}^{I-2} \frac{l\,(x_i + (l-1)\,\beta x_i) + (I-1)\,(\alpha x_i - c)}{I} + \frac{T\,((I-1)\,(x_i\,(1+\alpha) - c + (I-2)\,\beta x_i))}{I} = 0$$

Which implies

$$k(I,T) = \left\{ \frac{6c\,(I+T-1)}{3\,(2T+I-2) + 6\alpha\,(T+I-1) + 2\beta\,(6-6T-5I+I^2+3TI)} \right\}$$

For $k \in \left[\frac{c}{1+\alpha}, \frac{c}{\alpha}\right]$, equation (1) is:

$$\sum_{l=0}^{I-1} \frac{Tl\,(x_i\,(1+\alpha) - c + (l-1)\,\beta x_i) + (I-1-l)\,(\alpha x_i - c)}{I} = 0$$

which implies

$$k(I,T) = \left\{ \frac{3c\,(1+T)}{3T + 3\alpha\,(1+T) - 2\beta T\,(2-I)} \right\}$$ ■

Convergence to the static solution $(T = 1)$

Both solutions for the static game are

$$k(I,1)=\left\{\begin{array}{cc}\frac{6c(I)}{3(I)+6\alpha(I)+2\beta\left(-2I+I^2\right)} & if\ k\in\left[\frac{c}{1+\alpha+\beta},\frac{c}{1+\alpha}\right]\\ \frac{6c}{3+6\alpha-2\beta(2-I)} & if\ k\in\left[\frac{c}{1+\alpha},\frac{c}{\alpha}\right]\end{array}\right\}\blacksquare$$

Convergence to the efficient Solution

The efficient solution is

$$k^{eff}=\left\{\frac{c}{\alpha+\beta\,(I-2)+1}\right\}$$

and the limit for the relevant interval is:

$$\lim_{T\to\infty}k(I,T)=\left\{\frac{c}{1+\alpha+\beta\,(I-2)}\right\}\blacksquare$$

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